Pressure in Wave Propagation

For a plane progressive wave, the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (1)}$$

The velocity of particle is given by

$$U = \frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (2)}$$

The strain in the medium is dy/dx, which means that if dy/dx is positive, it represents a region of rarefaction. If dy/dx is $-ve$, it represents the area of compression.

By definition, the E_y of elasticity of the medium

$$K = \frac{\text{Change in Pressure}}{\text{Volume strain}} = \frac{-dP}{(dy/dx)} \quad \text{--- (3)}$$

or $\quad dP = -K(dy/dx)$

or $\quad dP = K\left(-\frac{dy}{dx}\right) \quad \text{--- (4)}$

This means that if dy/dx is negative, then dP $+ve$ is the area of compression. If dy/dx is $+ve$, then dP is $-ve$, that is, it is the region of rarefaction.

Differentiating equation (1) with respect to x we get

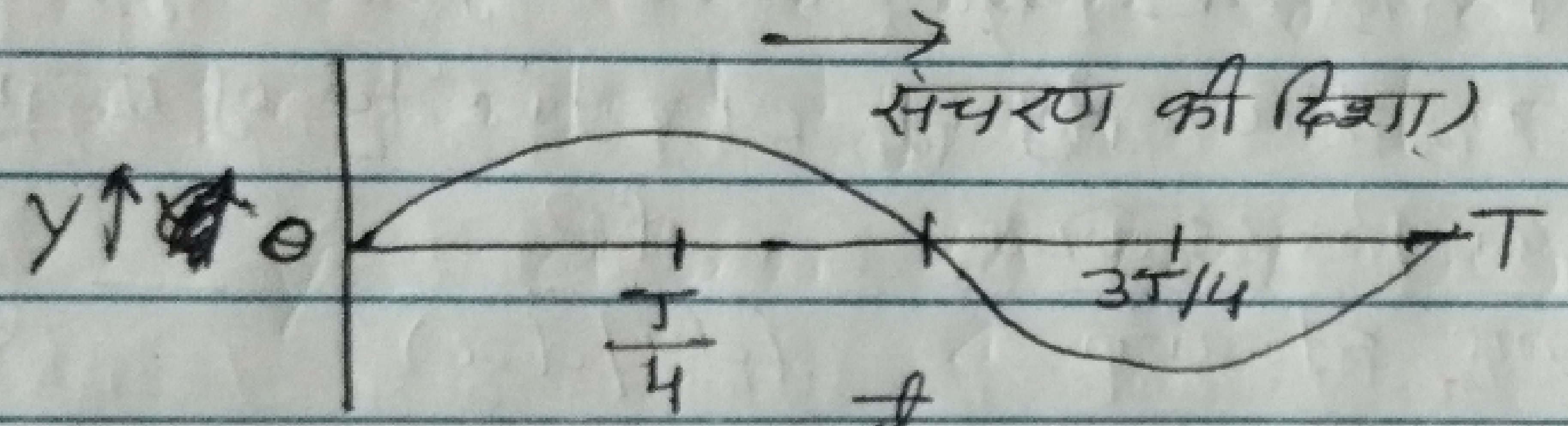
$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (5)}$$

From equation (4), we have

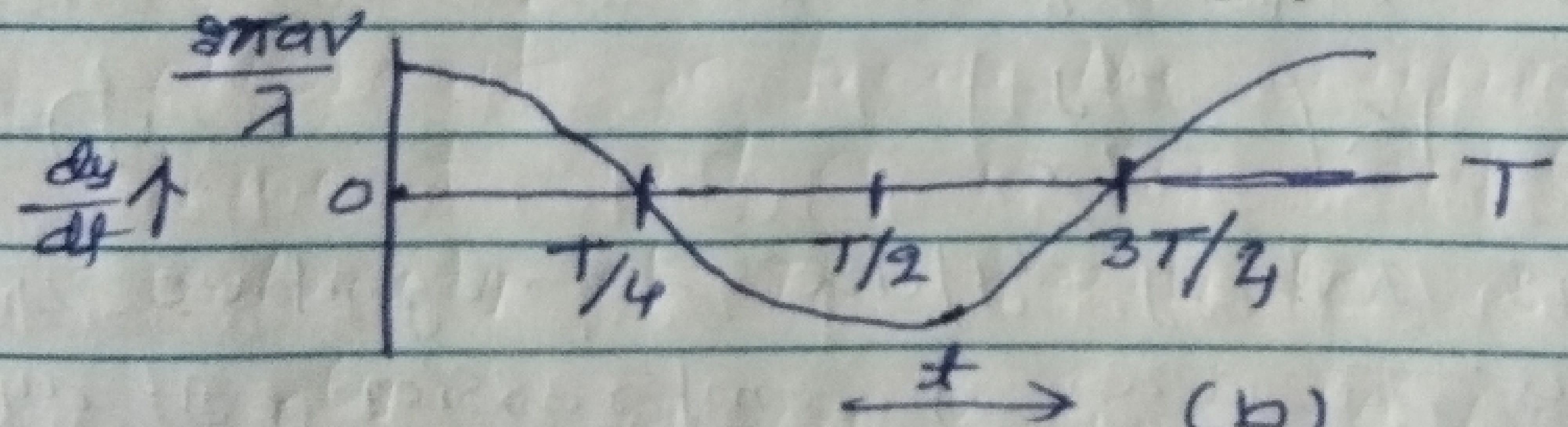
or $dP = K(-dy/dx)$

or $dP = \frac{2\pi K a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$ —— (6)

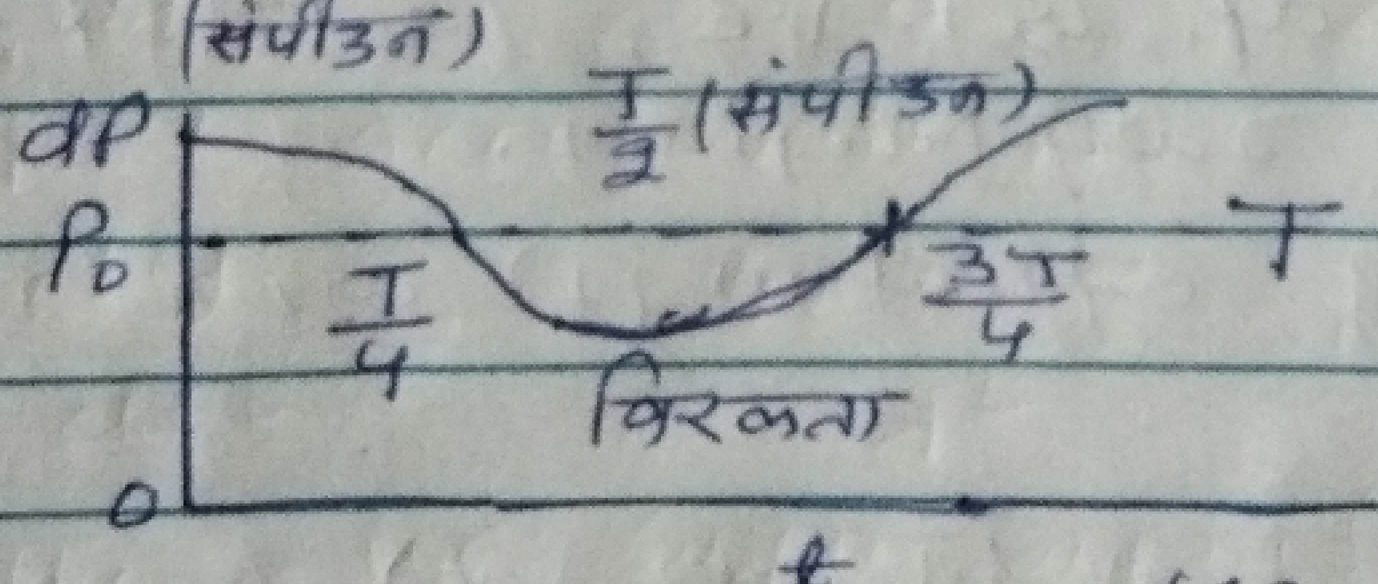
The graphs of the change in displacement, velocity and pressure are given in fig. where dP corresponds to a change in pressure, P_0 is the normal pressure of the medium when the wave does not propagate through it.



(a)



(b)



(c)

Figure

Energy in Wave Propagation

When a wave travels Through by a medium which means That transfer of energy take Place from one Place to another Place in The direction of Propagation of The wave.

If V is the velocity of the wave, Therefore the energy transferred Per Second is equal to The energy Contained by The Particles of The Medium in length V. The energy of a Moving wave at any instant is Partly Potential and Partly Kinetic.

Since the Particles of the medium Perform Simple harmonic Motion, The Kinetic energy is maximum at the Mean Position, where the displacement is zero and the Particle velocity of Maximum.

Potential energy is maximum at The extreme Position where displacement is maximum and Particle velocity is zero. This means That at the mean Position, The entire energy of The wave is Kinetic (Potential energy is zero) and at the extreme Position the entire energy of the wave is Potential (Kinetic energy is zero).

Note That, it can be said That There is no transfer of The medium in The direction of Propagation of The Progressive wave but There is only a transfer of energy in The direction of Propagation of The wave.

Let, a Progressive wave of wavelength λ travelling with a velocity v along The +ve direction of X-axis is represented by The expression.

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Here, y is The displacement at a Point distant x from the origin at any time t and a is the amplitude. The velocity of the Particle is given as

$$\frac{dy}{dt} = a\,2\pi \frac{v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

And acceleration

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x)$$

The -ve sign signifies That The acceleration of The Particles is always directed towards the mean Position.